STEPS IN GRA

1. Data Pre-processing.
2. Normalizing.
3. Deviation Sequence.
4. Grey Relational Coefficient.
5. Grey Relational Grade.

DATA PRE-PROCESSING & NORMALIZING

- The data to be used in Grey analysis must be preprocessed into quantitative indices for normalizing raw data for another analysis.
- Preprocessing raw data is a process of converting an original sequence into a decimal sequence between 0.00 and 1.00 for comparison.
- If the expected data sequence is of the form "**Higher-the-better**", then the original sequence can be normalized as,

$$x_i^*(k) = \frac{x_i^0(k) - \min x_i^0(k)}{\max x_i^0(k) - \min x_i^0(k)}$$

DATA PRE-PROCESSING & NORMALIZING

4. where $x^{o}_{i}(k)$ is the original sequence, $x^{*}_{i}(k)$ the sequence after the data preprocessing, max $x^{o}_{i}(k)$ the largest value of $x^{o}_{i}(k)$, and min $x^{o}_{i}(k)$ imply the smallest value of $x^{o}_{i}(k)$.

5. When the form "**Smaller-the-better**" becomes the expected value of the data sequence, the original sequence can be normalized as,

$$x_i^*(k) = \frac{\max x_i^0(k) - x_i^0(k)}{\max x_i^0(k) - \min x_i^0(k)}$$

DEVIATION SEQUENCE

1. The deviation sequence of the reference sequence is given by,

$$\Delta_{0i}(k) = \left\| x_0^*(k) - x_i^*(k) \right\|$$

$$\Delta_{\max} = \max_{\forall j \varepsilon i} \max_{\forall k} \left\| x_0^*(k) - x_j^*(k) \right\|,$$

$$\Delta_{\min} = \min_{\forall j \varepsilon i} \min_{\forall k} \left\| x_0^*(k) - x_j^*(k) \right\|$$

2. ζ is distinguishing or identification coefficient: $\zeta \in [0, 1]$. $\zeta = 0.5$ is generally used

GREY RELATIONAL COEFFICIENT

1. Grey relational coefficient is calculated to express the relationship between the ideal and actual normalized experimental results. Thus the grey relational coefficient can be expressed as,

$$\zeta_i(k) = \frac{\Delta_{\min} + \zeta.\Delta_{\max}}{\Delta_{0i}(k) + \zeta.\Delta_{\max}}$$

where Δ_{oi} (k) is the deviation sequence of the reference sequence